Vertical projection from the top of a tower

Motion in a straight line

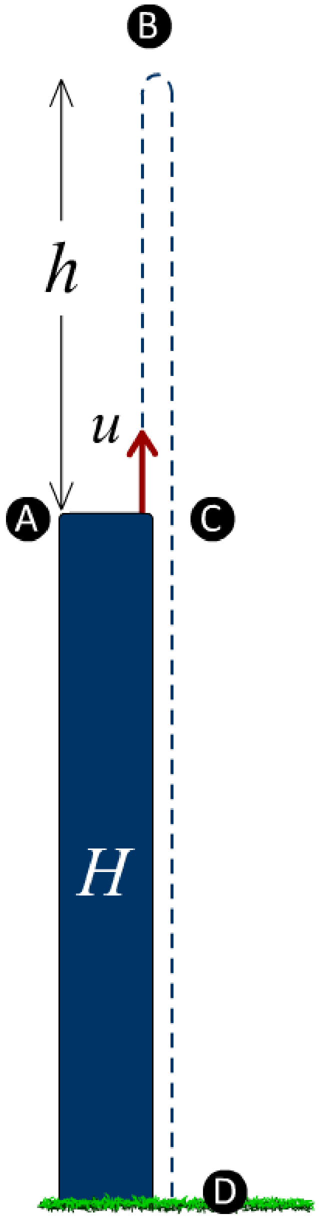
Consider a body projected vertically up from the top of a tower of height H. Let the initial velocity of the body be u. Velocity of the body decreases during ascent, becomes momentarily zero at the highest point and then increases in the downward direction during descent due to acceleration due to gravity (acting downward throughout its motion)

Important points

- Its initial velocity (u) is NOT zero
- Acceleration is due to gravity (9.8 ms^{-2} and downwards)
- TOA is not equal to TOD
- Displacement is NOT zero

Quantities to analyze/determine

- TOA, TOD, TOF
- Maximum height reached from the top of the tower
- Final velocity
- Average velocity



Maximum height (h) from the top of the tower

It is the height at which the velocity of the body becomes zero momentarily.

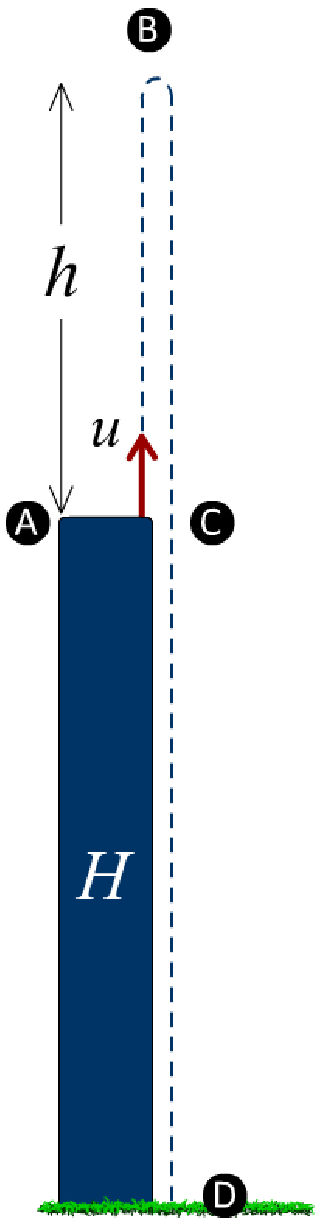
Instantaneous velocity, as a function of displacement, for a body moving with constant acceleration is given by

$$v^2 - u^2 = 2aS$$

Considering the motion from A to B, we get v = 0 and a = - g (as it is downwards) and S = h we get

$$0 - u^2 = 2(-g)(h)$$

$$\boxed{h = \frac{u^2}{2g}}$$



Time of ascent (TOA)

It is the time at which the velocity of the body becomes zero momentarily.

Instantaneous velocity, as a function of time, for a body moving with constant acceleration is given by

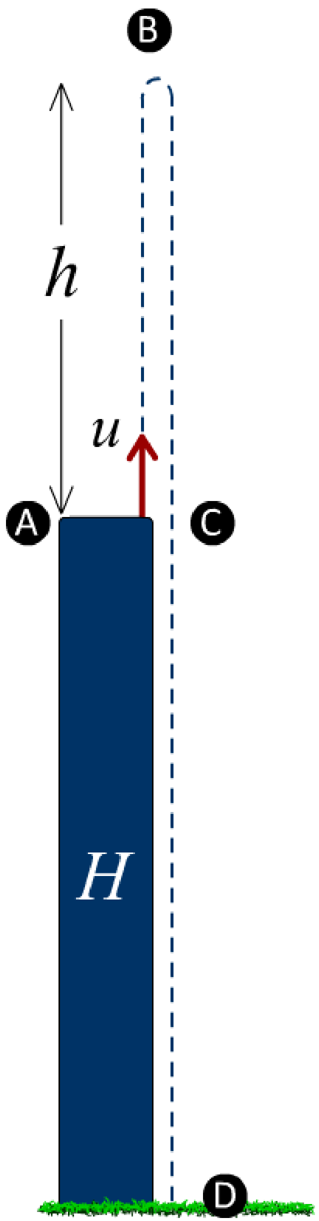
$$v = u + at$$

Considering the motion from A to B, a = - g and v = 0 at the highest point therefore

$$0 = u + (-g)t$$

$$\Rightarrow \quad t = \frac{u}{g}$$

$$\boxed{TOA = \frac{u}{g}}$$



Time of descent (TOD)

TOD can be obtained using the relation for displacement as a function of time.

Considering the motion from B to D, height of the body from the ground is ($H + h$) therefore its displacement is - ($H + h$) .

$$S = ut + \frac{1}{2}at^2$$

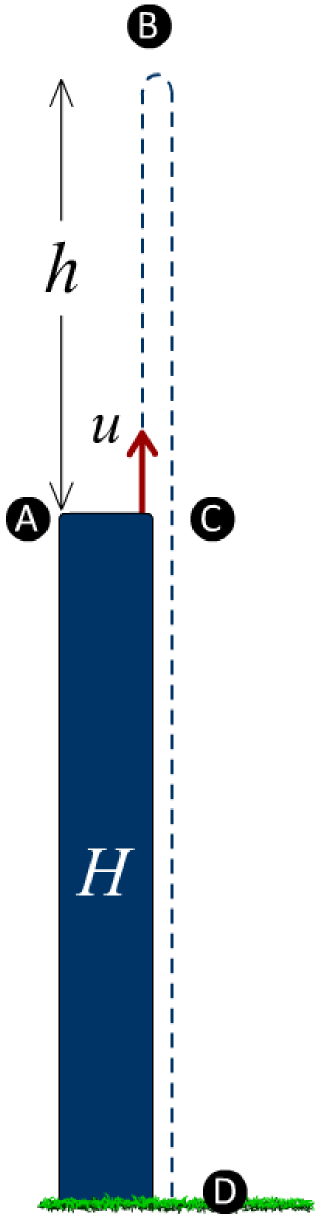
$$\Rightarrow \; -(H + h) = 0 + \frac{1}{2}(-g)t^2$$

$$\Rightarrow \; \left(H + \frac{u^2}{2g}\right) = \frac{1}{2}gt^2$$

$$\Rightarrow \; t^2 = \frac{2}{g}\left(H + \frac{u^2}{2g}\right)$$

$$\Rightarrow \; t = \sqrt{\frac{2}{g}\left(H + \frac{u^2}{2g}\right)}$$

$$\boxed{TOD = \sqrt{\frac{2}{g}\left(H + \frac{u^2}{2g}\right)}}$$



Final velocity (v)

From the highest point, the body reaches the ground as a freely falling body. Therefore displacement is - ($H + h$) and initial velocity for descent as zero. Using equation of motion for velocities from B to D, we get

$$v^2 - u^2 = 2aS$$

$$v^2 - 0 = 2(-g)(-(H + h))$$

$$v^2 = 2g(H + h)$$

Using the value of h obtained earlier we get

$$v^2 = 2g\left(H + \frac{u^2}{2g}\right)$$

$$v = \sqrt{2g\left(H + \frac{u^2}{2g}\right)}$$

Projection from the top of a tower

Consider a body projected vertically up with an initial velocity u, from the top of tower of height H. The body ascends to a height h from the top of a tower and then descend to the ground. In this case

- Net displacement is not zero
- Displacement during ascent is $+h$
- Displacement during descent if $-(h + H)$
- Net displacement is $-H$
- $TOA \neq TOD$